

4.

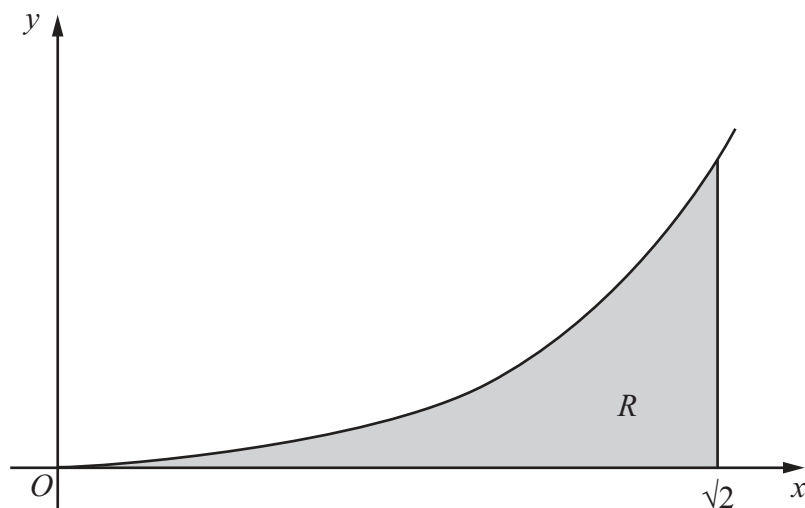


Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geq 0$. The finite region R , shown shaded in Figure 2, is bounded by the curve, the x -axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln(x^2 + 2)$.

| | | | | | |
|-----|---|----------------------|----------------------|-----------------------|------------|
| x | 0 | $\frac{\sqrt{2}}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{3\sqrt{2}}{4}$ | $\sqrt{2}$ |
| y | 0 | | 0.3240 | | 3.9210 |

(a) Complete the table above giving the missing values of y to 4 decimal places. (2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places. (3)

(c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2} \int_2^4 (u - 2) \ln u \, du \quad (4)$$

(d) Hence, or otherwise, find the exact area of R . (6)



6. With respect to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet and find the position vector of their point of intersection A . (6)

(b) Find, to the nearest 0.1° , the acute angle between l_1 and l_2 . (3)

The point B has position vector $\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$.

(c) Show that B lies on l_1 . (1)

(d) Find the shortest distance from B to the line l_2 , giving your answer to 3 significant figures. (4)



7.

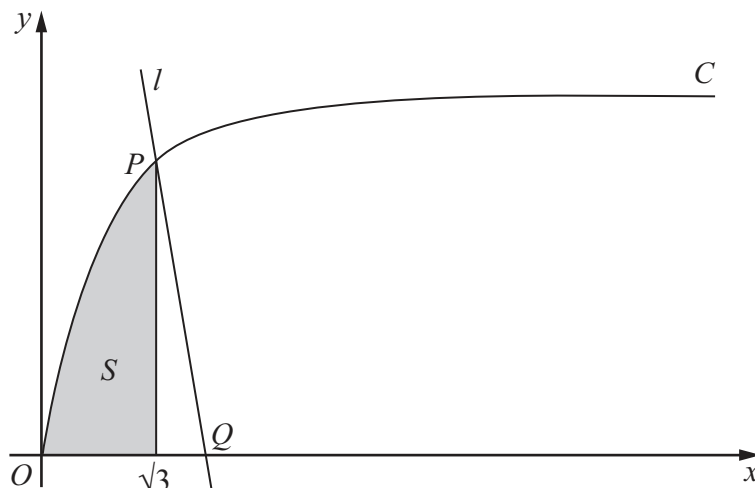


Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan \theta, \quad y = \sin \theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(a) Find the value of θ at the point P . (2)

The line l is a normal to C at P . The normal cuts the x -axis at the point Q .

(b) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k . (6)

The finite shaded region S shown in Figure 3 is bounded by the curve C , the line $x = \sqrt{3}$ and the x -axis. This shaded region is rotated through 2π radians about the x -axis to form a solid of revolution.

(c) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants. (7)



